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Project Abbie
Algorithm Development Update

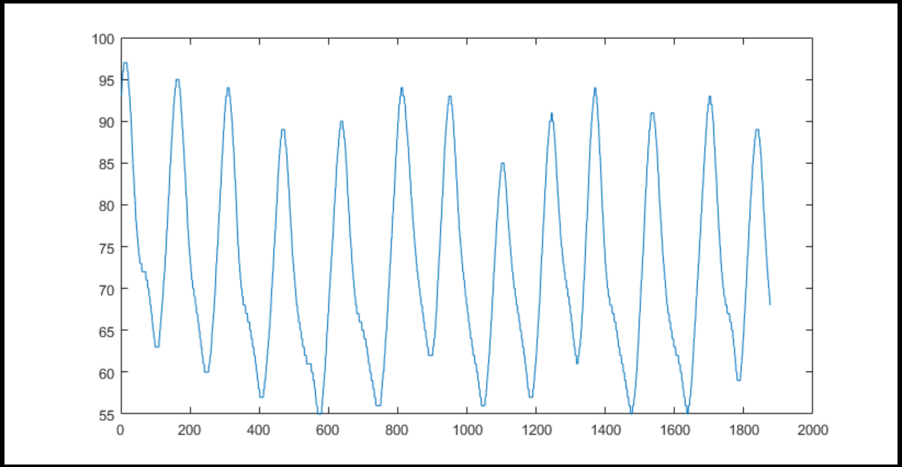
Frequency Domain Representations and Multifractal Analysis

SAHIL LOOMBA

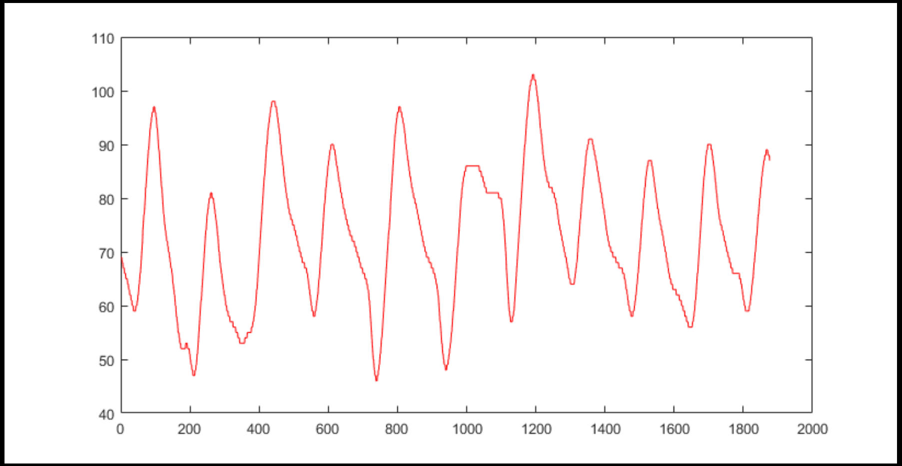
The story yet…

- Recall that we had a number of hand-designed features in our old feature space
  - Aggregate statistics and counts
  - Shape descriptors
  - Frequency descriptors
- Possibly correlated/related
  - Need for feature selection
- Big Question: What is the best representation of input data (feature space) from the POV of predicting HASS?
  - Representation learning = Machine learning!
  - But can we use domain knowledge to “add information” that helps separate the signal from the noise in a complex problem such as this, even before any “ML” is applied?

What does respiration look like?



Low HASS Breathing



High HASS Breathing

What does respiration look like?

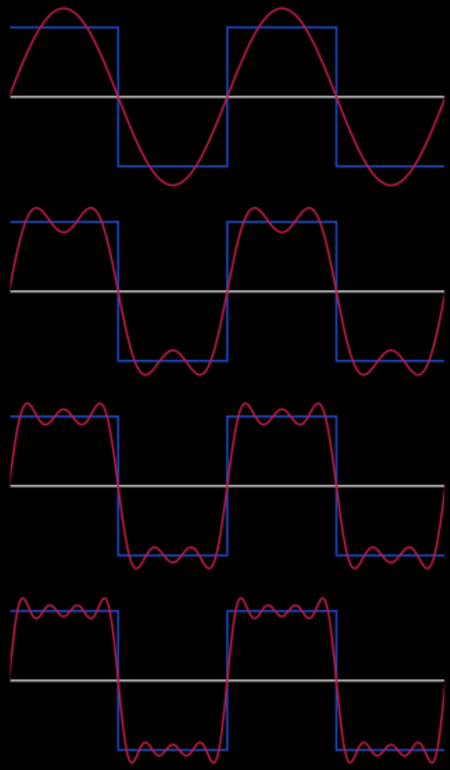
- (Almost) a periodic signal
- Hypothesis: Breathing can be constrained in the domain of periodic signals, and high HASS can be seen as an “aberration” within this domain
  - What counts as aberration? Depends…

Representing Periodic Signals

- Any set of signals is represented on some “basis” which they can “span”
- Say time signals $x(t)$ of length n have the basis $\mathbb{R}^n$
- We can change the representation by changing the “basis”
  - PCA finds a basis of orthogonal vectors that maximize data variance
  - For time signals, a basis corresponding to frequencies is a good idea
    - Inverse domains

Representing Periodic Signals

- Fourier Series: Express a periodic function as a weighted combination of sinusoids
  - Each parameter is uncorrelated!



$$x(t) = A_0 + \sum_{n=1}^{N} A_n \sin\left(\frac{2\pi}{N} nft + B_n\right)$$

Fourier Series Coefficients
Basis: $\mathbb{R} \times \mathbb{R} \times \mathbb{R}^{+N} \times \mathbb{R}^{N}$
parameters: 2*(N+1)
But most higher order parameters would be 0s!
(In our analysis we curtail N to 8)

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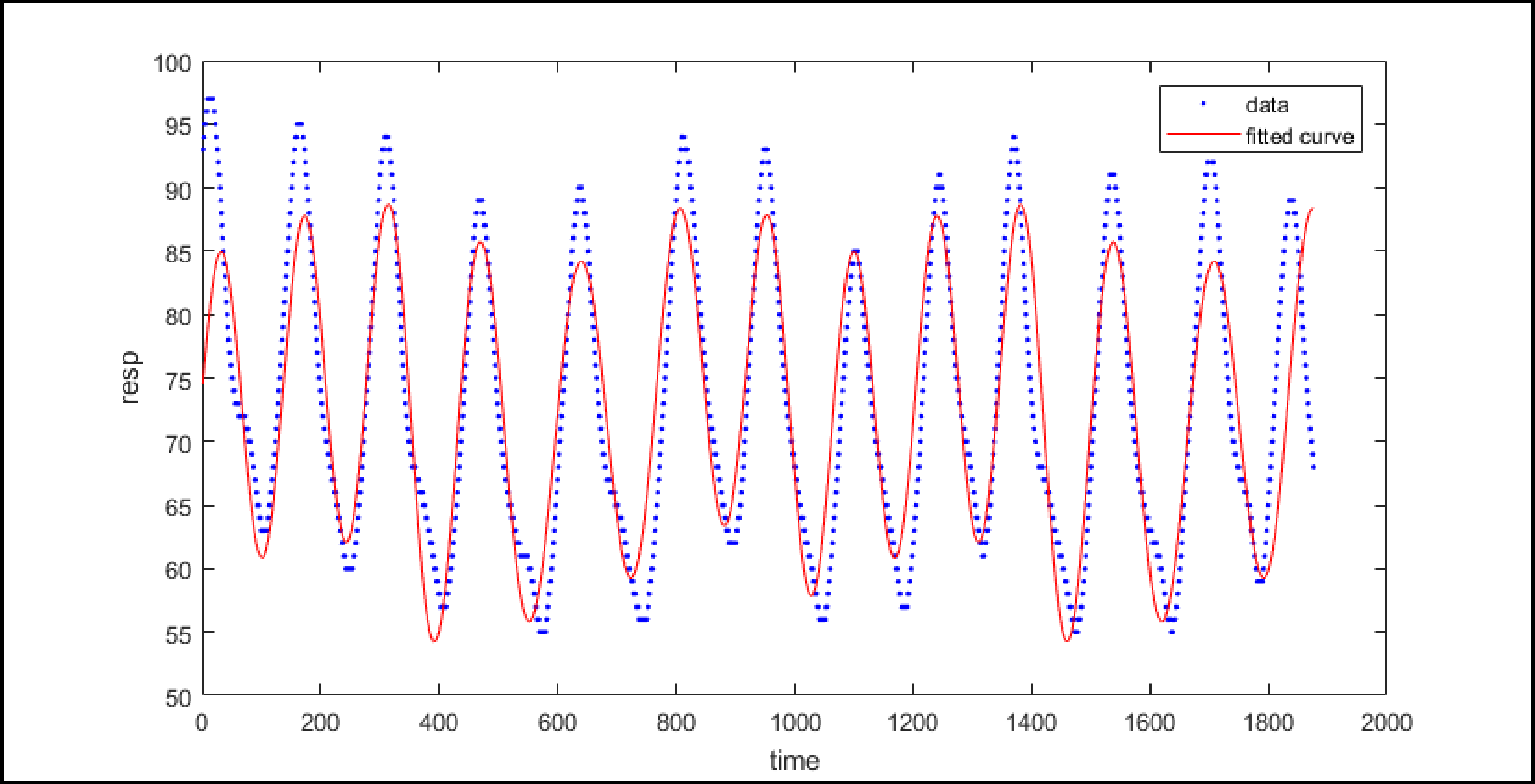
Representing Periodic Signals

- Fourier Series



Representing (A)periodic Signals

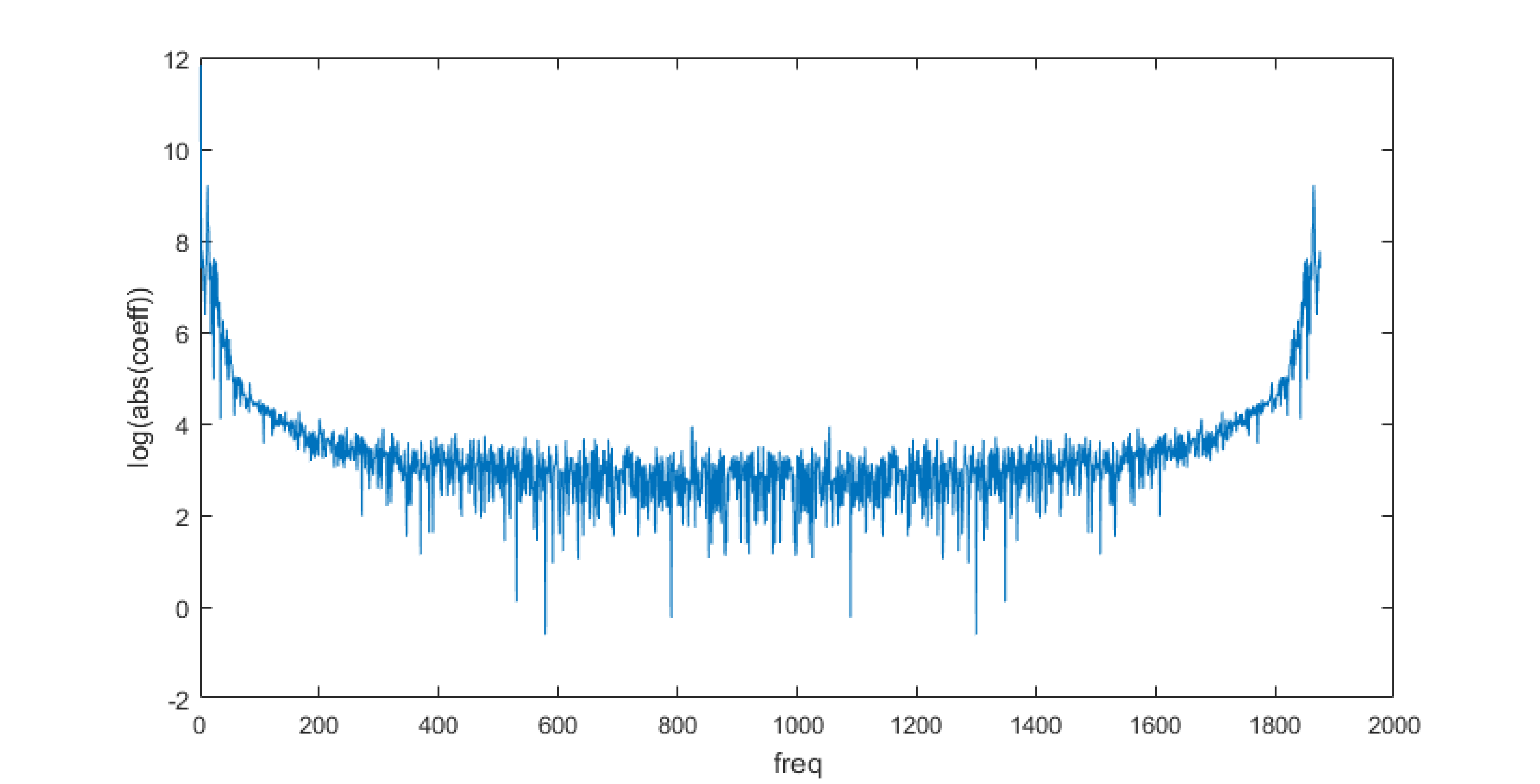
- (Discrete) Fourier Transform: Express an (a)periodic (discretely sampled) function as a weighted combination of complex sinusoids

$$x(t) = A_0 + \sum_{n=1}^{N-1} A_n \left(\cos\left(\frac{2\pi}{N} nt\right) + i \sin\left(\frac{2\pi}{N} nt\right)\right)$$

Discrete Fourier Transform
Basis: $\mathbb{R}^{+N}$
parameters: N
But most parameters would be 0s! (sparse)

Representing (A)periodic Signals

- (Discrete) Fourier Transform:



Representing (A)periodic Signals

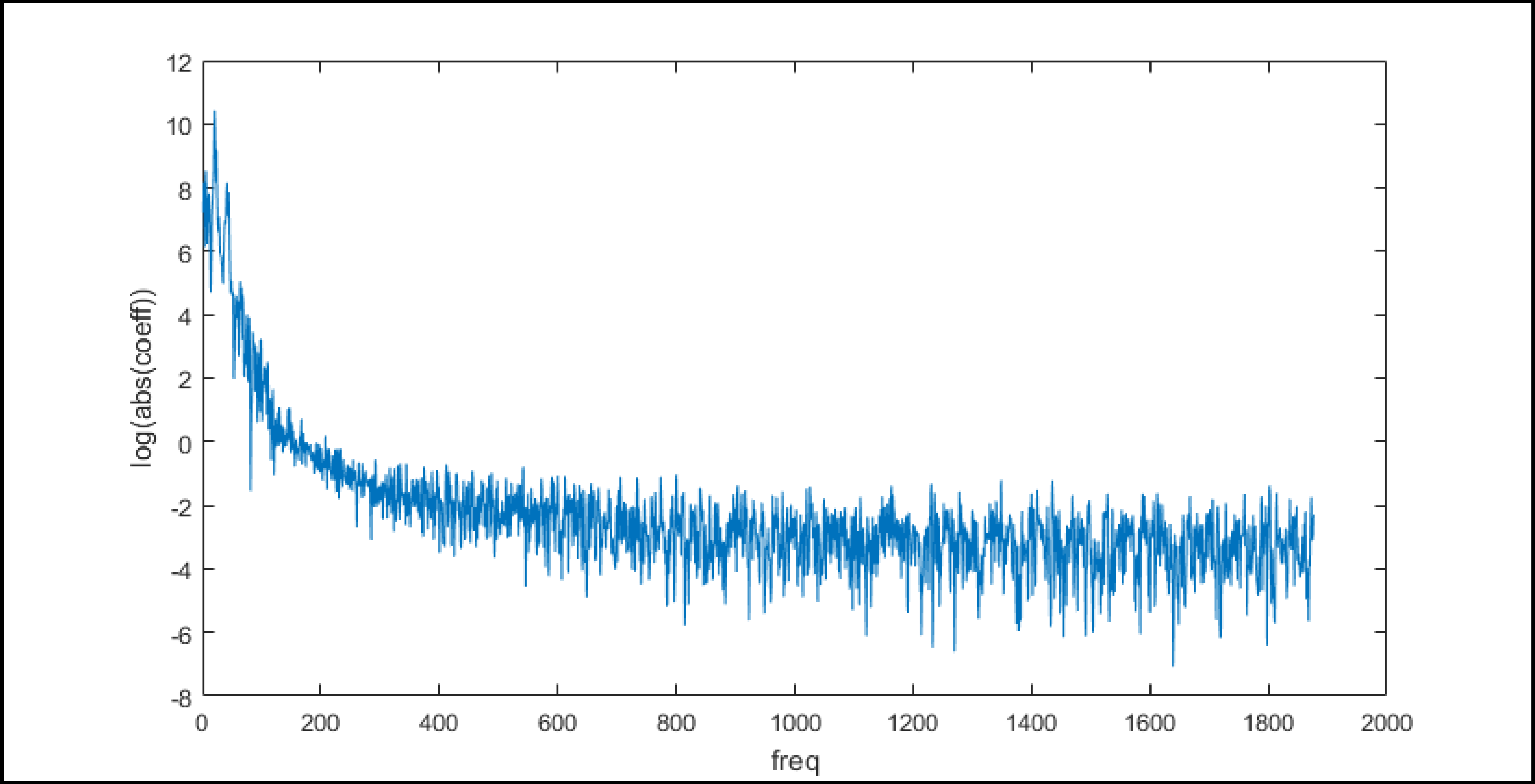
which may not be sampled regularly

- Least Squares DFT: Express an (a)periodic (discretely sampled) function as a weighted combination of complex sinusoids while ensuring least squares fit
  - Better than DFT for long-gapped/irregularly sampled data

Representing (A)periodic Signals

which may not be sampled regularly

- Least Squares DFT:



Representing (A)periodic Signals

whose spectra might be noisy

- Welch's Method on DFT: Subdivide signal into smaller windows and compute the averaged DFT over frequency "bins"
  - Better than DFT when spectrum is noisy (loss of frequency resolution but also reduction in noise)
  - Also called Short-Time-Fourier-Transform (STFT)

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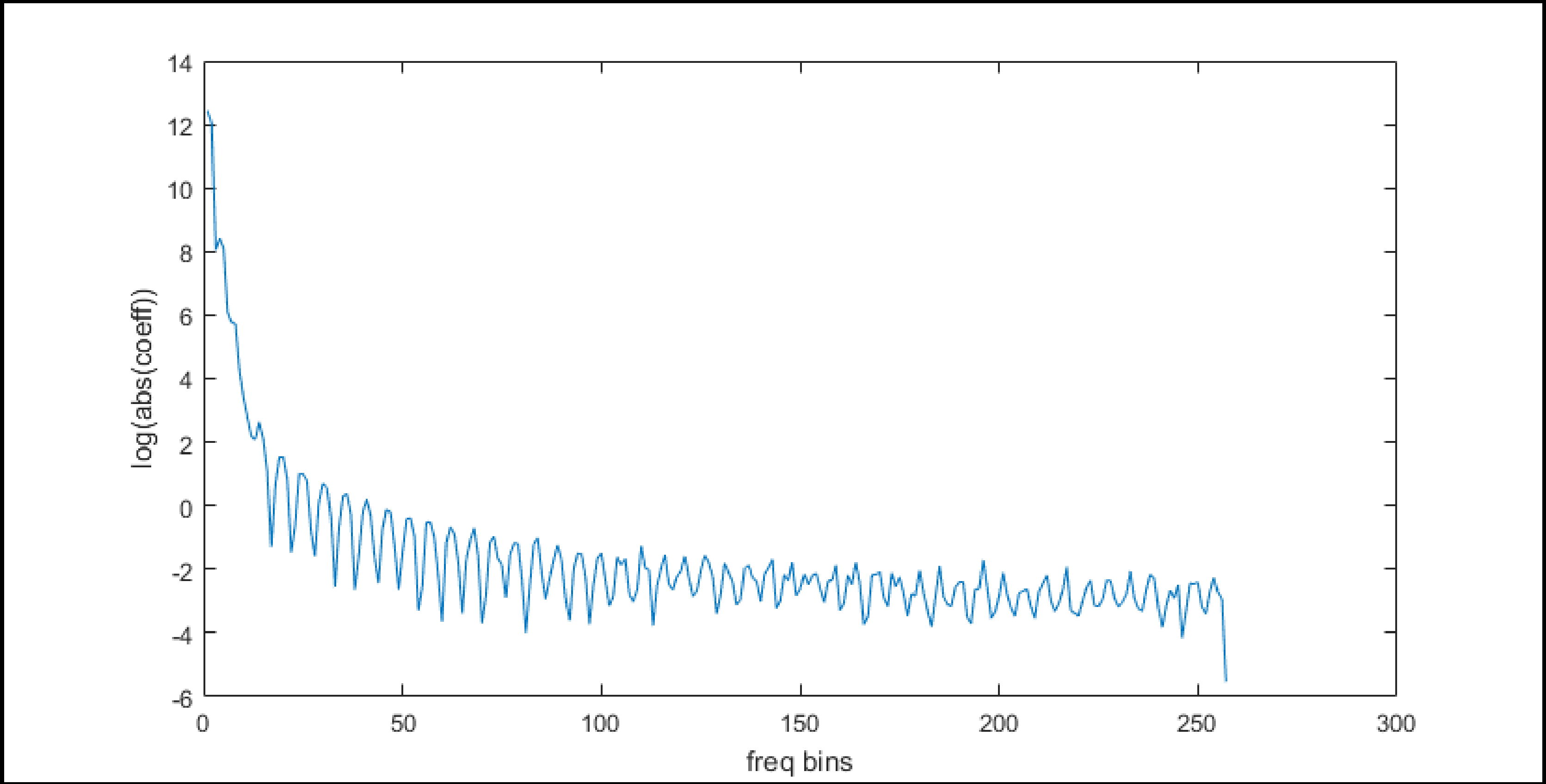
Representing (A)periodic Signals

whose spectra might be noisy

- Welch's Method on DFT:



Question

- Can we impose some stricter conditions to make the representation sparser (more efficient)?
  – Hope: sparsity causes only the *most* significant information to be preserved in the transformation
- Answer: Discrete Cosine Transform
  – Imposes a certain “boundary condition” on DFT that extends the signal in an even-periodic fashion
  – Usually sparser than DFT

Representing (A)periodic Signals

with more parsimony?

- (Discrete) Cosine Transform: Express an (a)periodic (discretely sampled) function as a weighted combination of real cosines

$$x(t) = A_0 + \sum_{n=1}^{N-1} A_n \cos\left(\frac{\pi}{N} n \left(t + \frac{1}{2}\right)\right)$$

Discrete Cosine Transform

Basis: $\mathbb{R}^{+N}$

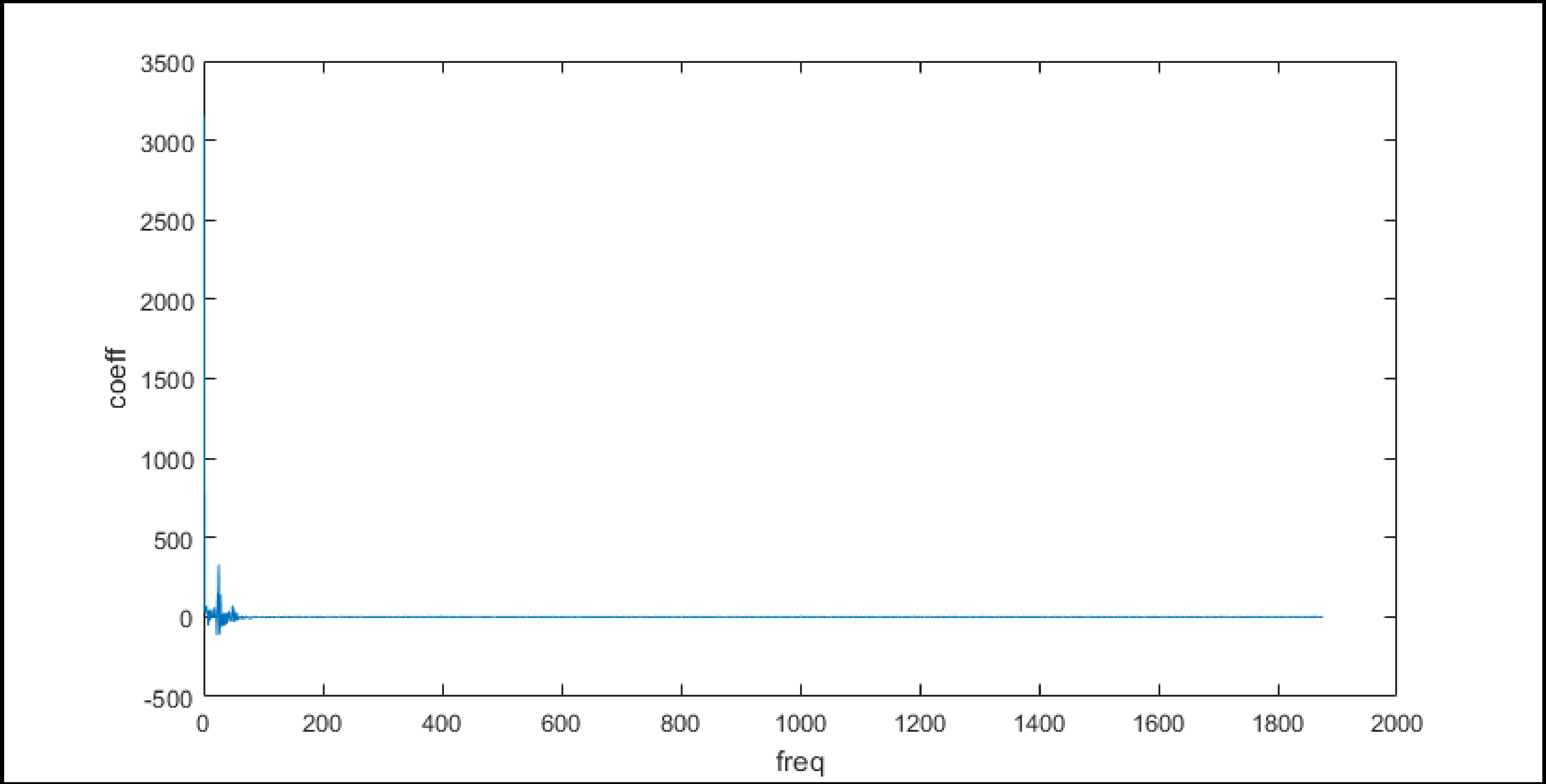
parameters: N

But almost all parameters would be 0s! (very sparse)

Representing (A)periodic Signals

with more parsimony?

- (Discrete) Cosine Transform:



Question

- Can we find a basis "better" than sinusoids?
  - Hope: although the signal is periodic, there could be a more effective representation if the basis also reflects some notion of "shape" of the signal
- Answer: Discrete Wavelet Transform
  - Uses a "wavelet" of a certain shape and expresses signal as a weighted combination of parametrized instances of the wavelets
  - Since we incorporate shape within the basis itself, a more compact representation
  - Captures both frequency and time regularity (trade-off), unlike Fourier analysis

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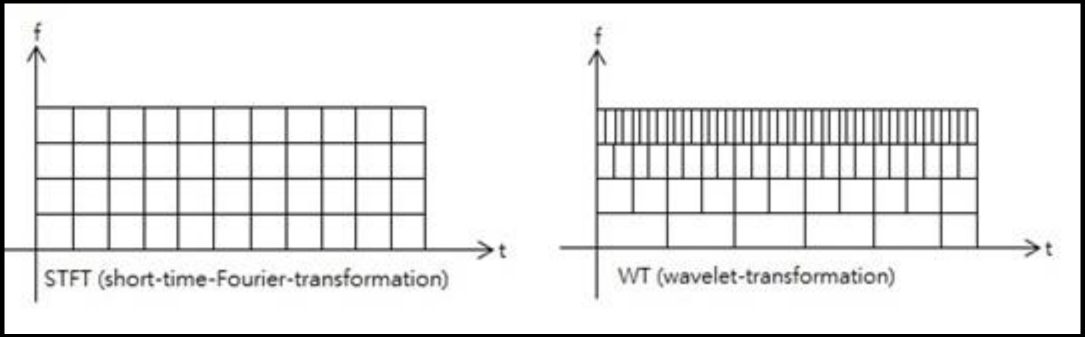
Representing (A)periodic Signals

with both temporal and frequency resolution

- (Discrete) Wavelet Transform: Express an (a)periodic (discretely sampled) function as a weighted combination of wavelets

$$x(t) = \sum_{m\in\mathbb{Z}} \sum_{n\in\mathbb{Z}} \langle x, \psi_{m,n} \rangle \, \psi_{m,n}(t) \qquad \psi_{m,n}(t) = \frac{1}{\sqrt{a^m}} \psi\left(\frac{t - nb}{a^m}\right)$$

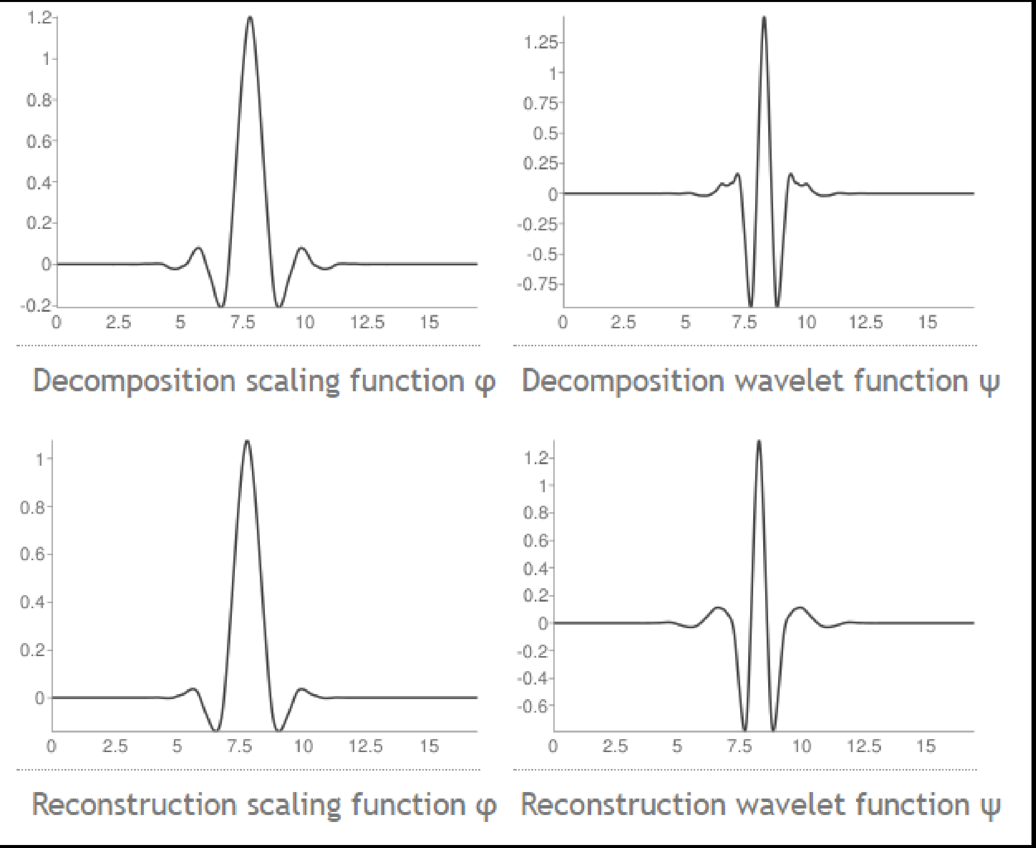
  – STFT is a special case where wavelet is $e^{-2\pi i t}$



Representing (A)periodic Signals

with both temporal and frequency resolution

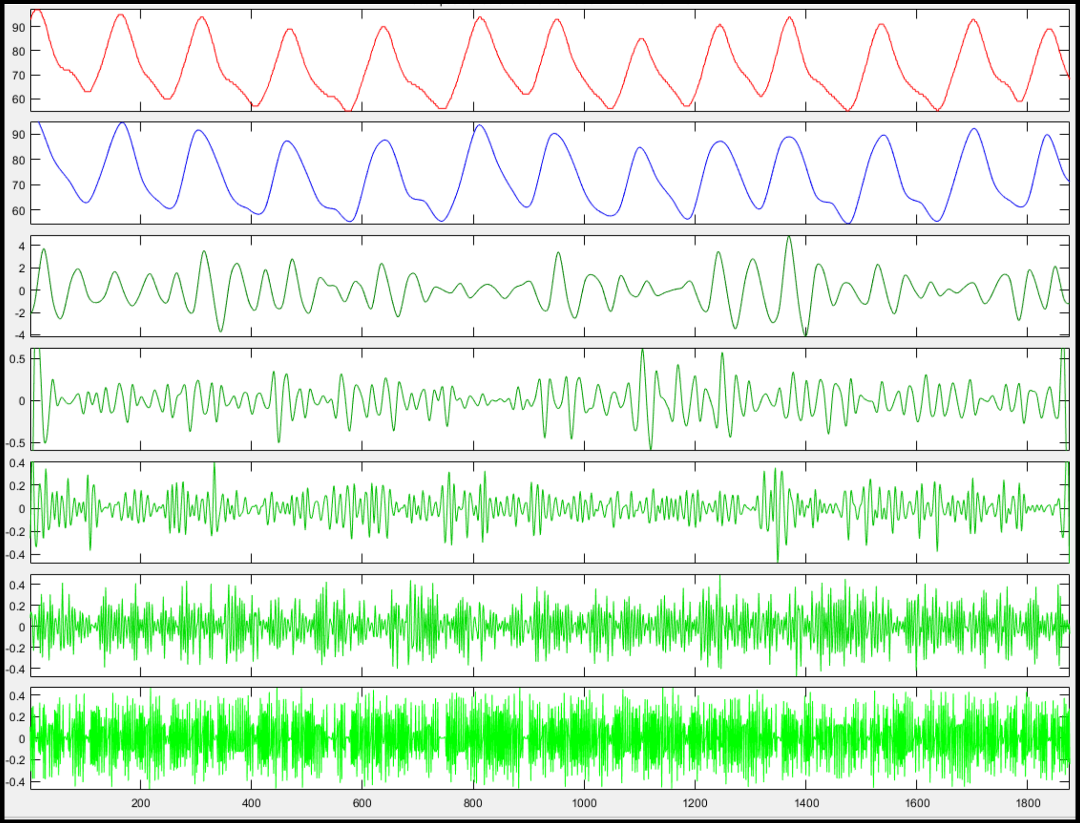
- What wavelet to choose?
  - Multiresolution analysis needs a biorthogonal filter (for some important mathematical properties to be satisfied)
  - We choose bior6.8 (notice its shape)
- Fix a=2, b=1



Representing (A)periodic Signals

with both temporal and frequency resolution

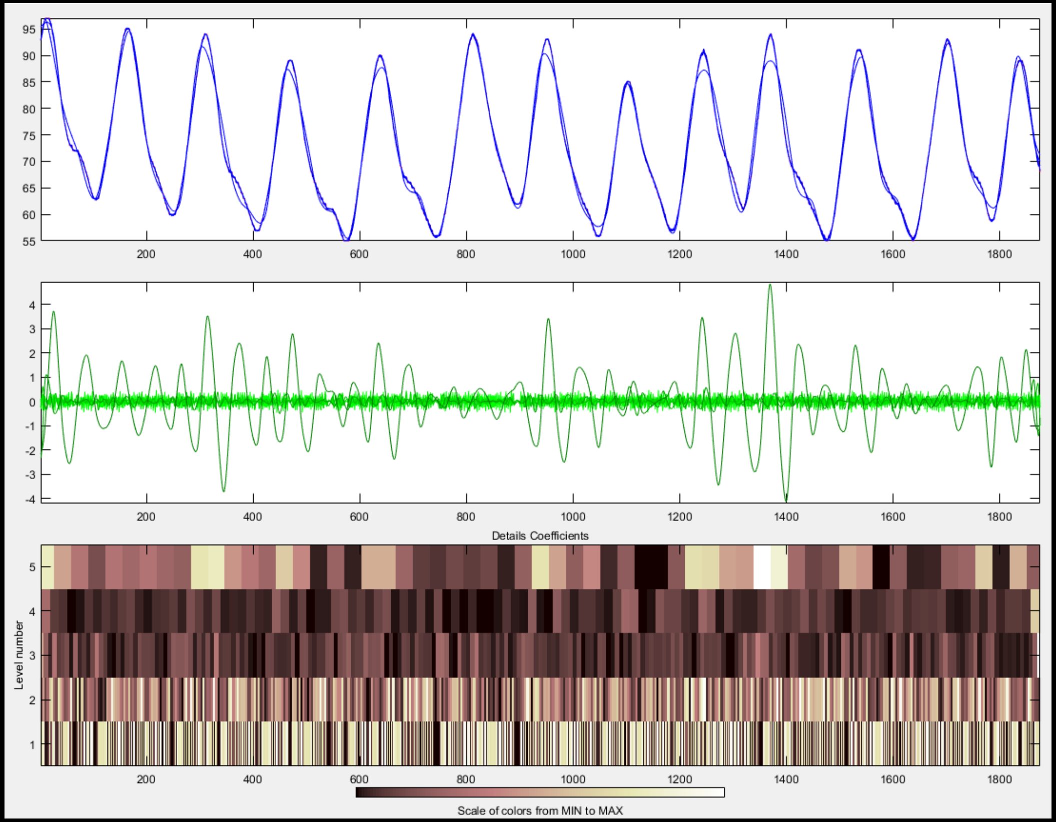
- Wavelet's scale allows us to play with time resolution



Representing (A)periodic Signals

with both temporal and frequency resolution

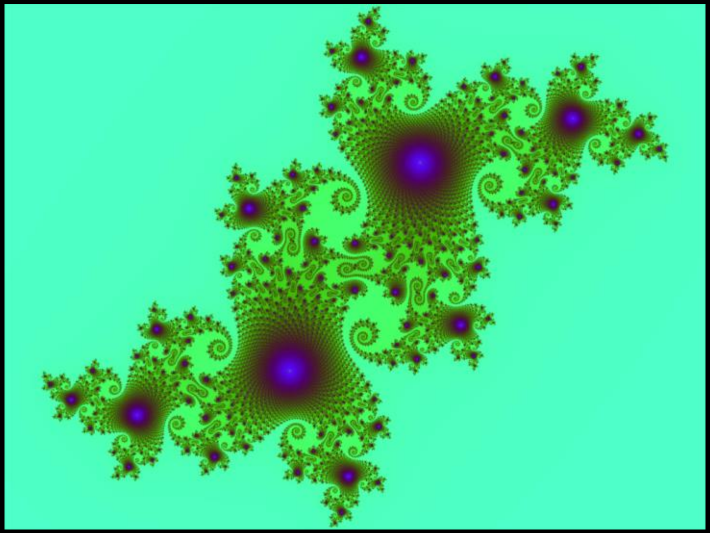
- Wavelet's scale allows us to play with time resolution

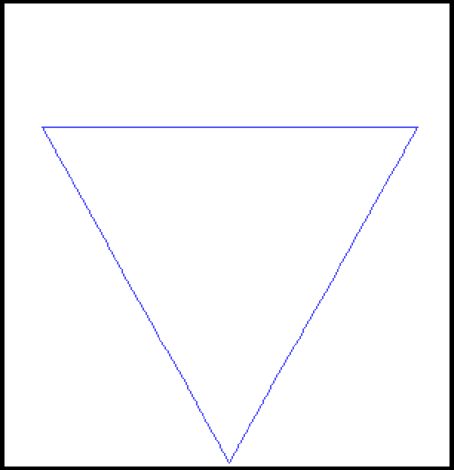


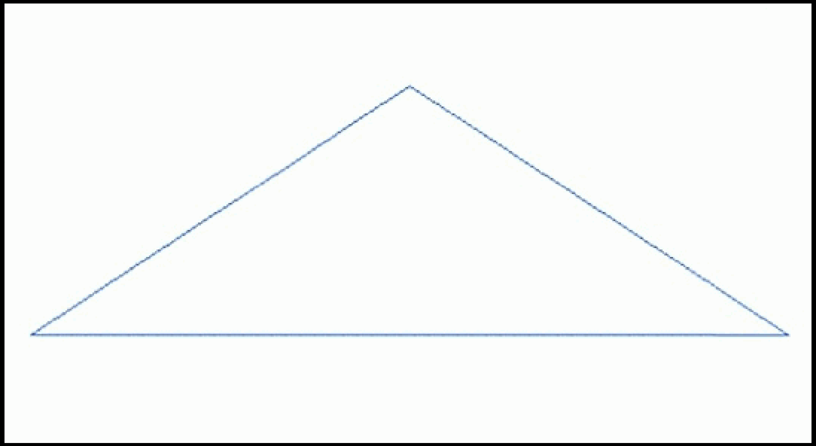
Question

- How to best use this multiresolution Wavelet representation?
- Answer: An aside first…

Aside: Fractals







Aside: Fractals

- Self-similar structures
- Fine or detailed structure at arbitrarily small scales
  - Leads to complex emergent properties
- Fracticality:
  - Monofractal: a single parameter describes the dynamics
  - Multifractal: a parameter spectrum describes the dynamics

Further aside: Power Law

- $f(x) = ax^{-k}$, where k is the scaling exponent
- Scale invariance: $f(ax) = c(ax)^{-k} = a^{-k}f(x)$
- Universality: Diverse systems with same "critical" scaling exponent can be shown to share same fundamental dynamics
- Common in physics, biology, social networks…
  - ECG, Neuronal spiking

Aside: Fractals

- Monofractal: a single power law relationship
  - between moments of some multiresolution quantity T and moments of scale

$$T(a,x)^q \sim a^{qk}$$

(for small appropriate ranges of scale a and moment q [-5:5 here])

- Multifractal: a scale dependent power law distribution
  - Can be expressed as a single power law distribution where instead of saying scale dependent (base in above expression), we say dependent on some function $\zeta(q)$ of moments (exponent in above expression)

$$T(a,x)^q \sim a^{\zeta(q)}$$

(Monofractal is just a special case where ζ is linear)

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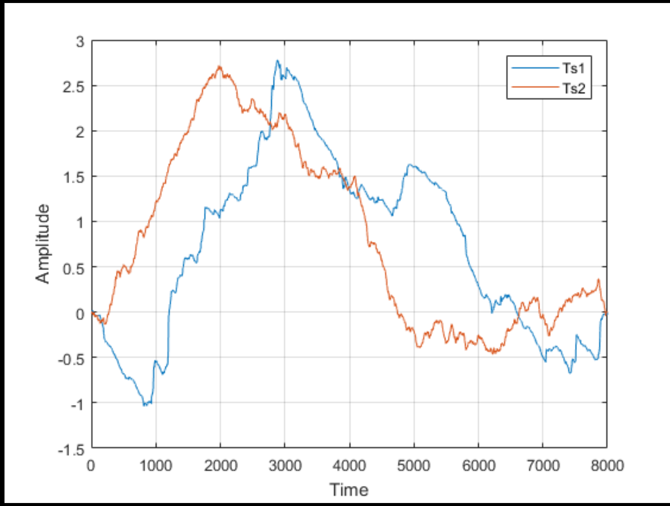
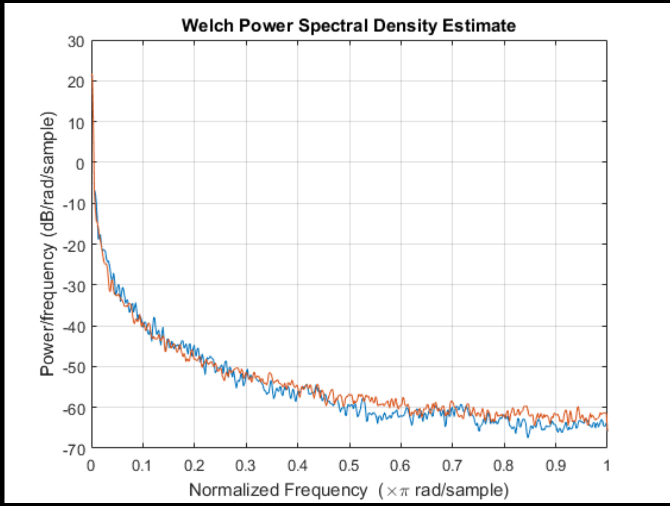
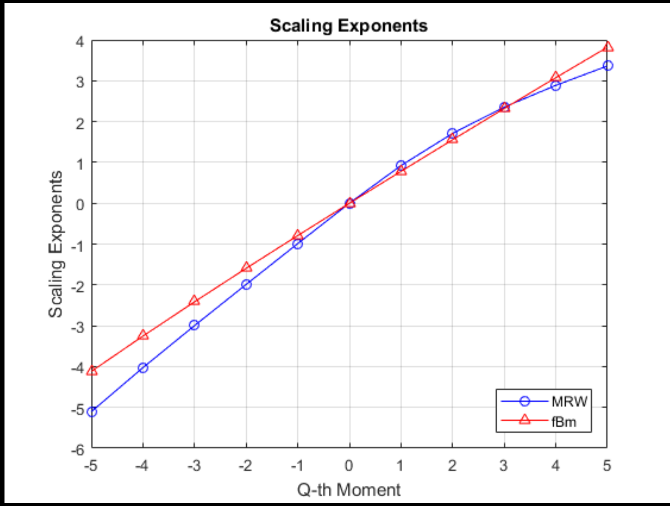
Aside: Fractals

- Multifracticality, therefore, is simply a departure from linearity of scaling exponents versus the qth moments







Question

- What multiresolution quantity to use as T?
- Answer is in the previous question: How to best use the multiresolution Wavelet representation?
- Let T(x,a) represent the wavelet coefficients of signal x at scale a
  – (Slightly incorrect; we use the average wavelet coefficient of the “leaders” of scale a, for some theoretical reasons…)

The story so far…

- Instead of working in the time domain, we obtain a frequency representation of the signal which can be more compact
  - **Spectral analysis:** Fourier series, DFT, LSSA, STFT, DCT
- We can work in a domain that does a trade-off in temporal and frequency resolutions and captures scale-free/self-similar properties of the signal
  - **Multifractal analysis:** DWT followed by estimation of scaling exponents

What does respiration look like?

- Hypothesis: Breathing can be constrained in the domain of periodic signals, and high HASS can be seen as an “aberration” within this domain
  - What counts as aberration?
- In the two closely defined representation philosophies:
  - **Spectral analysis:** an aberration is a dramatic change in the spectrum of the signal
    - In the spectral coefficient values
  - **Multifractal analysis:** an aberration is a dramatic change in the fractal spectrum of the signal
    - In the linearity of scaling exponents versus moment
    - Possibly: respiration becomes more multifractal during an asthmatic fit?

Experiments

- Data: We use a smaller respiration dataset (which was used for artifact detection) which has HASS labels from 5 through 14, collected from 8 subjects, after removing corrupt signals
- Code: Spectral analysis and wavelet toolboxes in MATLAB
- Big Question: What is the best representation of input data (feature space) from the POV of predicting HASS? We judge the quality of a representation visually for now
  - **Spectral analysis:** Do a PCA of high-D spectral coefficient space onto 2 dimensions
  - **Multifractal analysis:** Assume $\zeta(q)$ up to first 3 terms

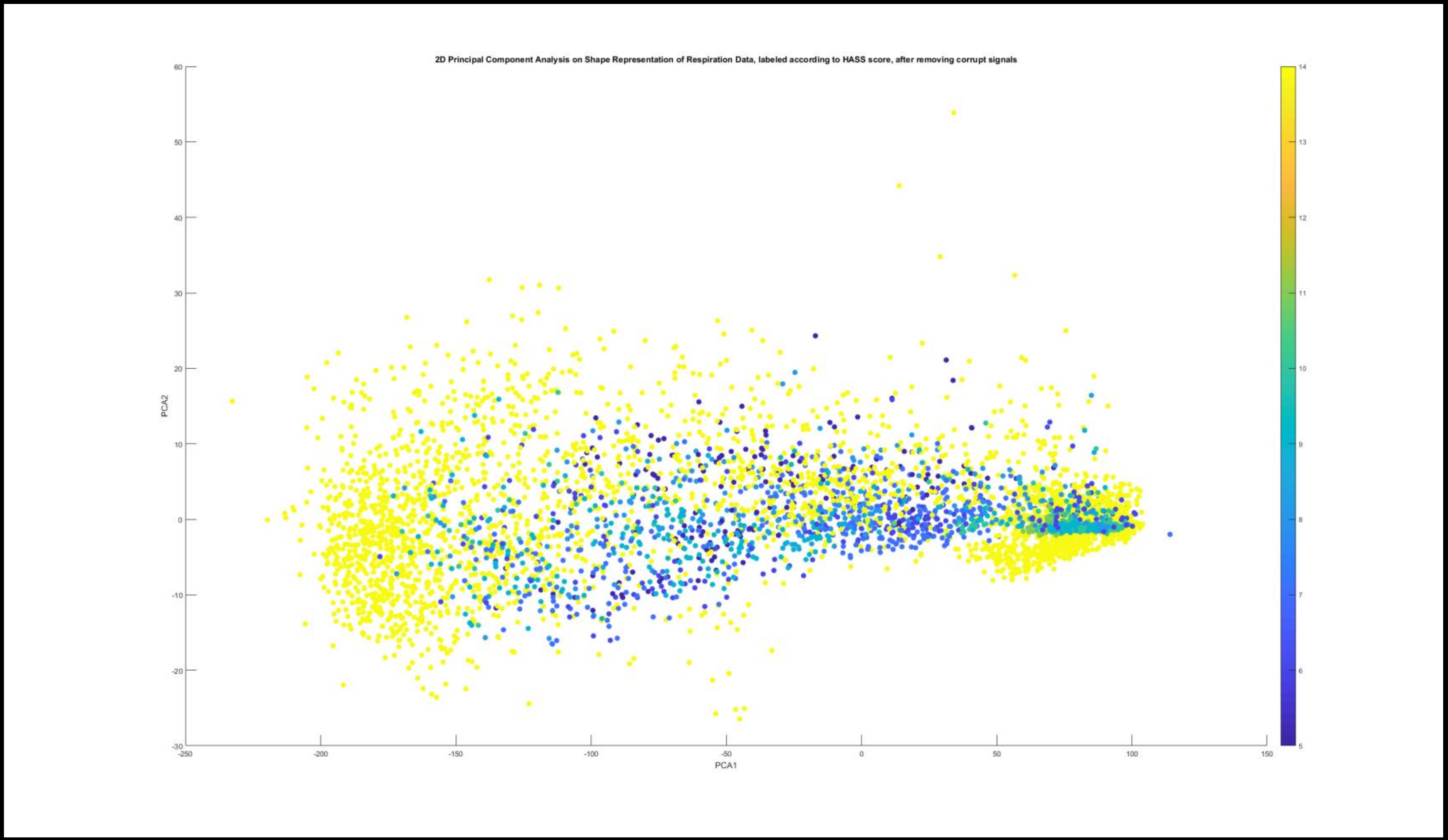
$$\zeta(q) = k_1 q + \frac{k_2 q^2}{2} + \frac{k_3 q^3}{6}$$

While k_1 (first cumulant) captures linearity, k_2 (second cumulant) captures extent of non-linearity (multifractality)

We do a 2 dimensional plot of the first two cumulants

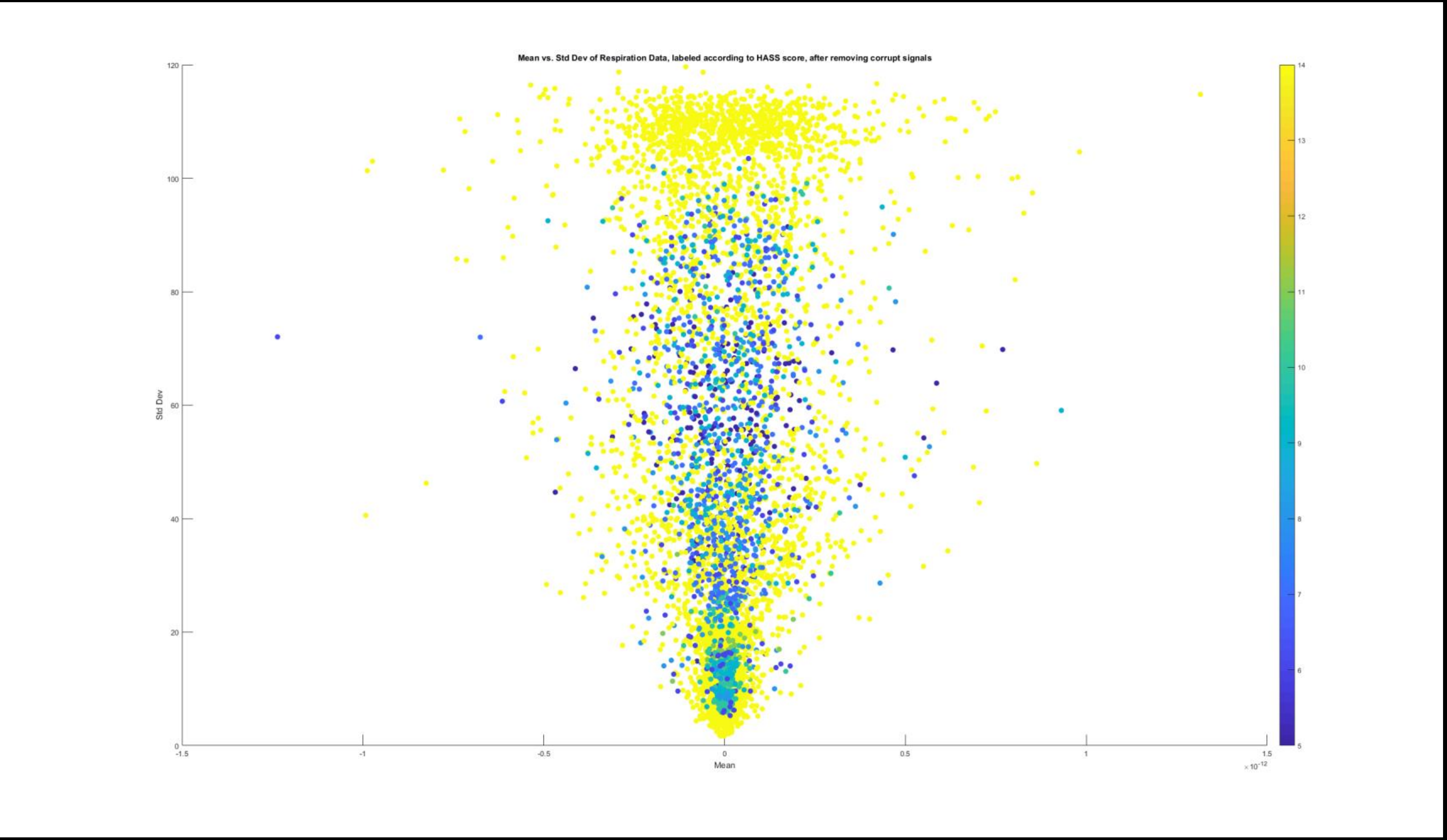
Experiments

PCA on old feature space's Shape Descriptors



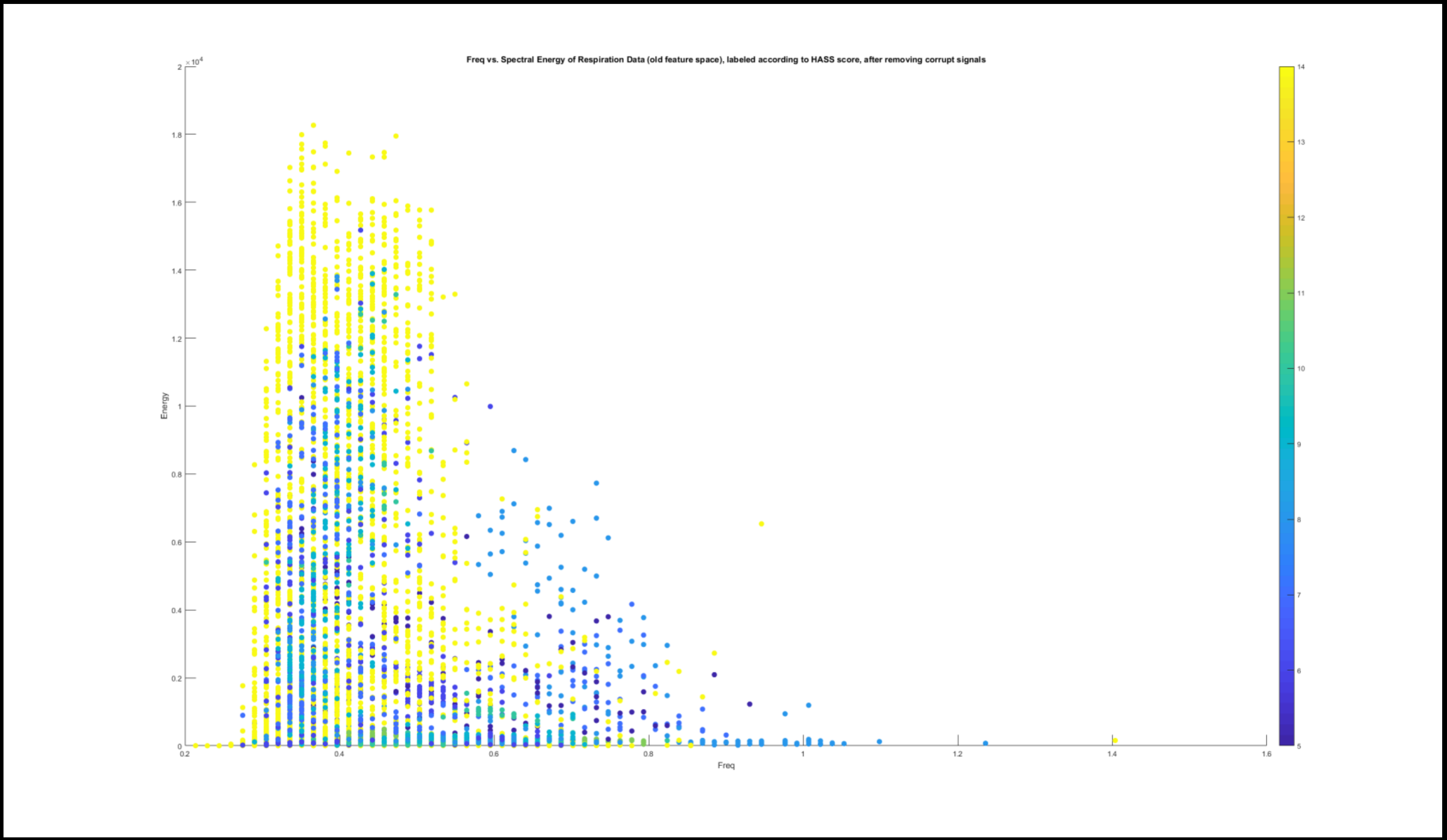
Experiments

Old feature space's Mean vs. Std Dev



Experiments

Old Feature Space's Freq vs. Energy



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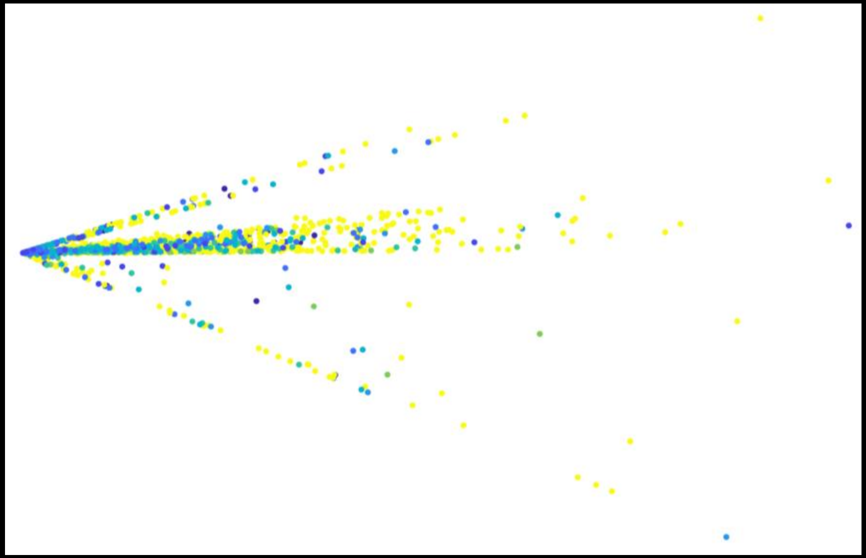
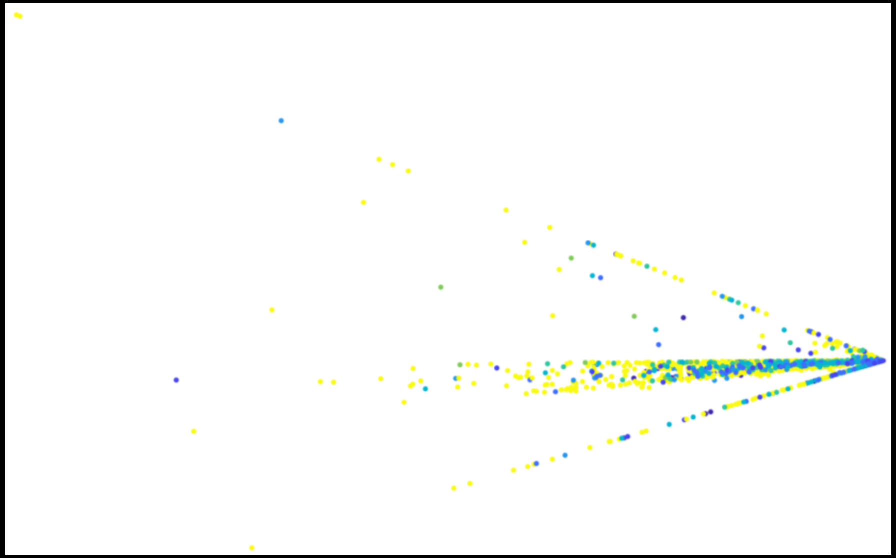
Experiments

PCA on Fourier Series Coefficients (upto 8)



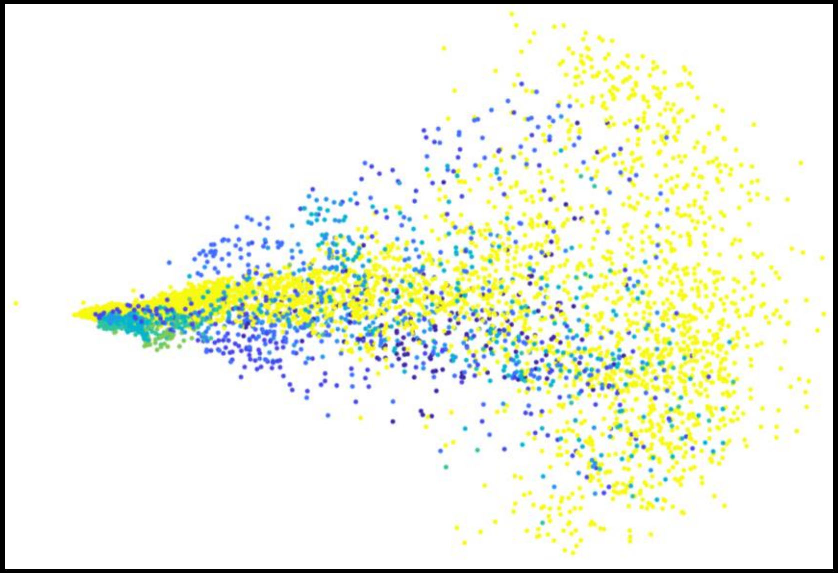
With DC component



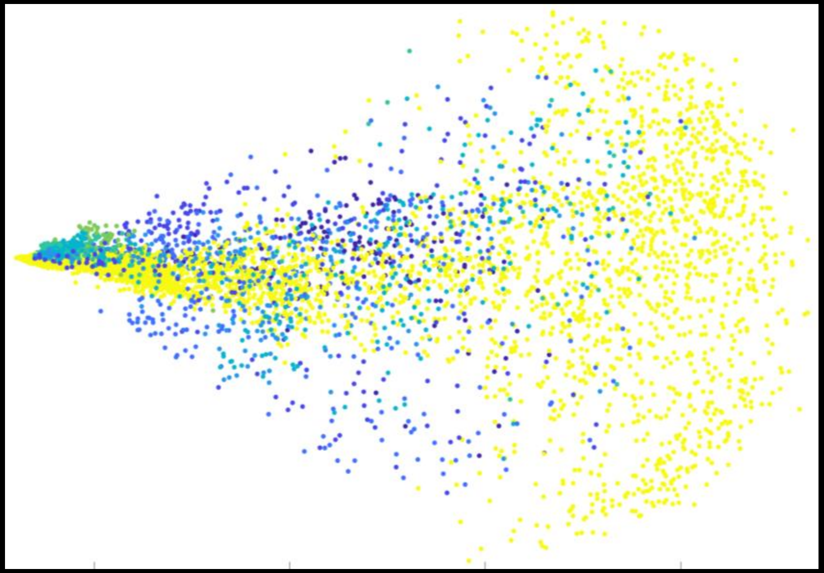
Without DC component

- Unclear visual distinction
- Too few parameters?
- DC seems to have no effect

Experiments
PCA on DFT Spectrum



With DC component

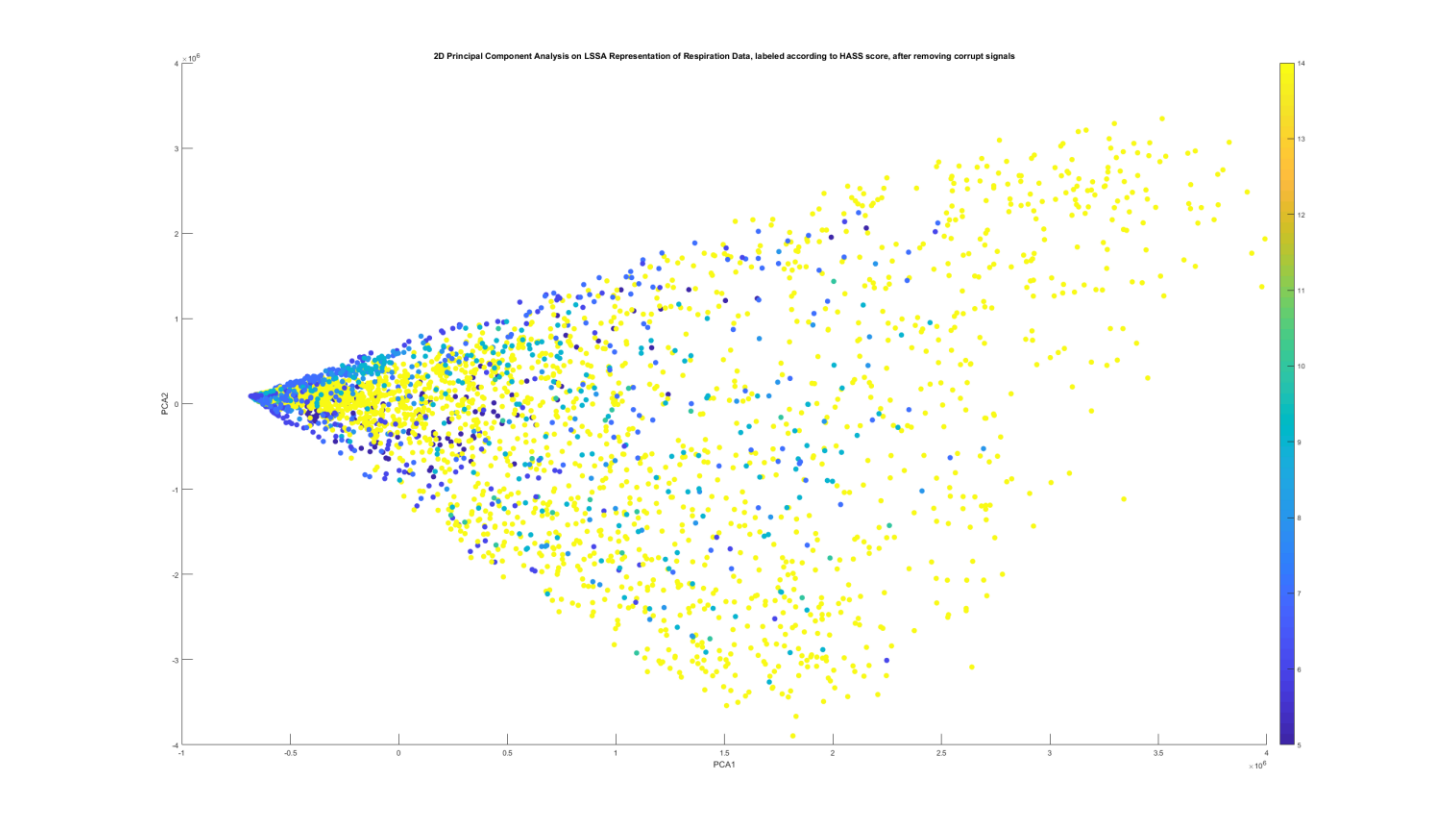


Without DC component

- Clearer visual distinction
- DC seems to have no effect

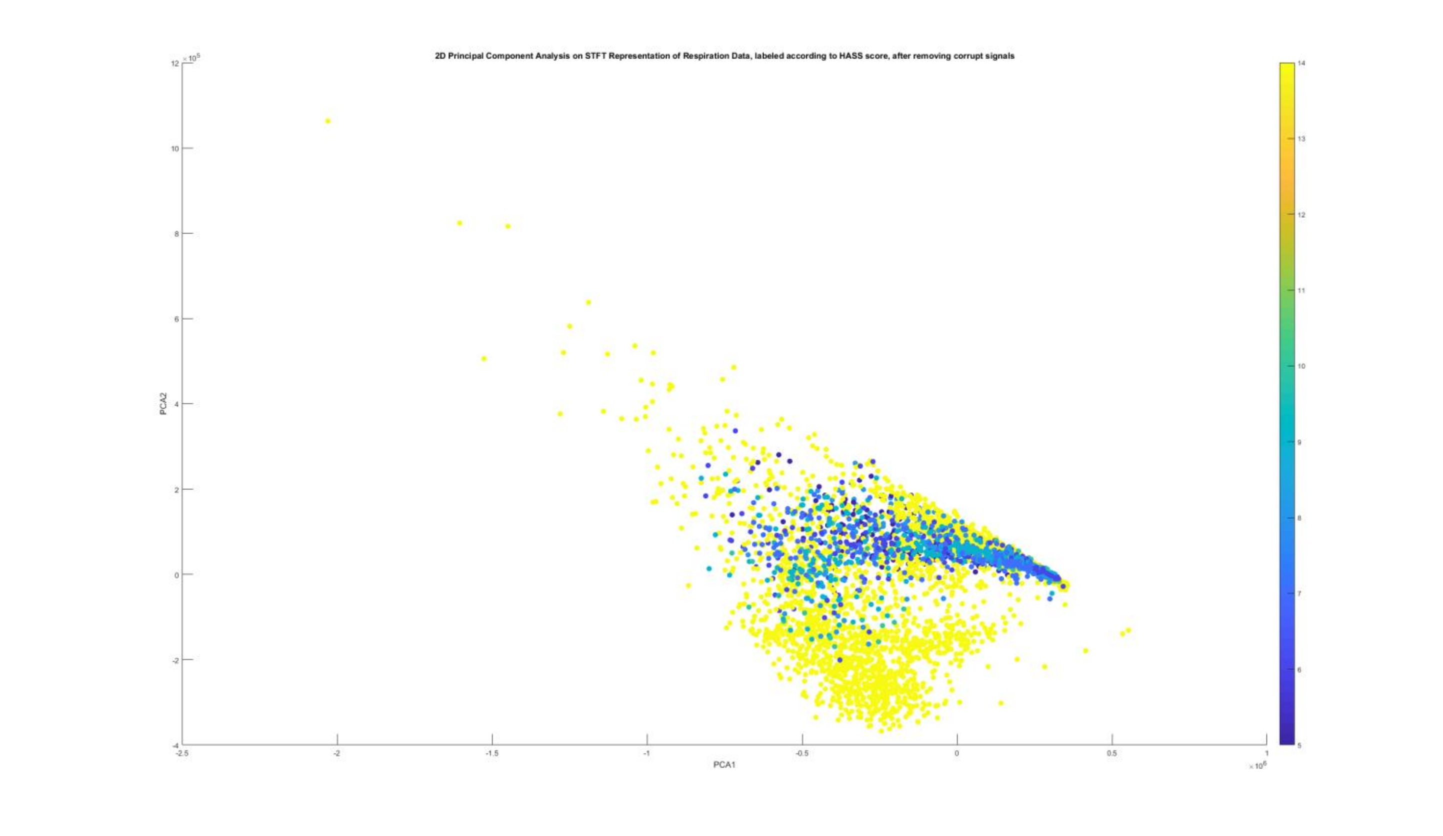
Experiments

PCA on LSSA Spectrum



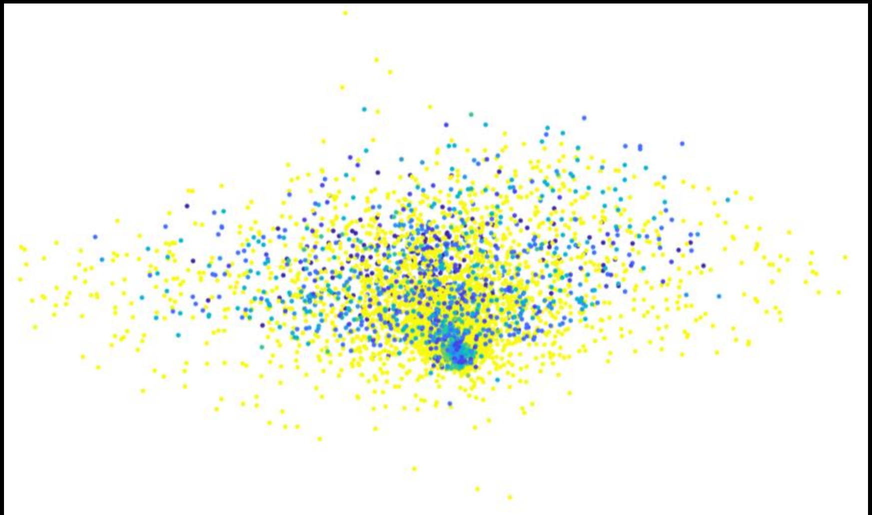
- Clearer visual distinction

Experiments
PCA on STFT Spectrum

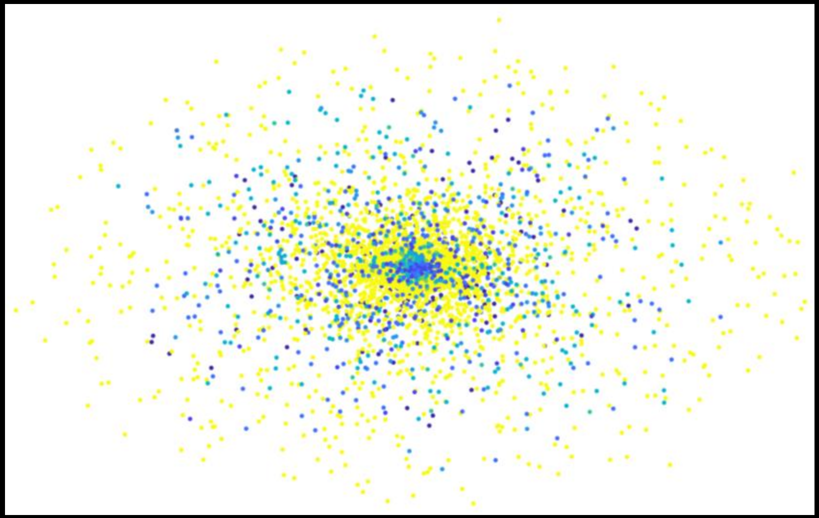


- Clearer visual distinction
- Losing frequency resolution reduces error in the frequency space

Experiments
PCA on DCT Spectrum



With DC component



Without DC component

- Visual distinction not as clear
- DCT is too sparse?

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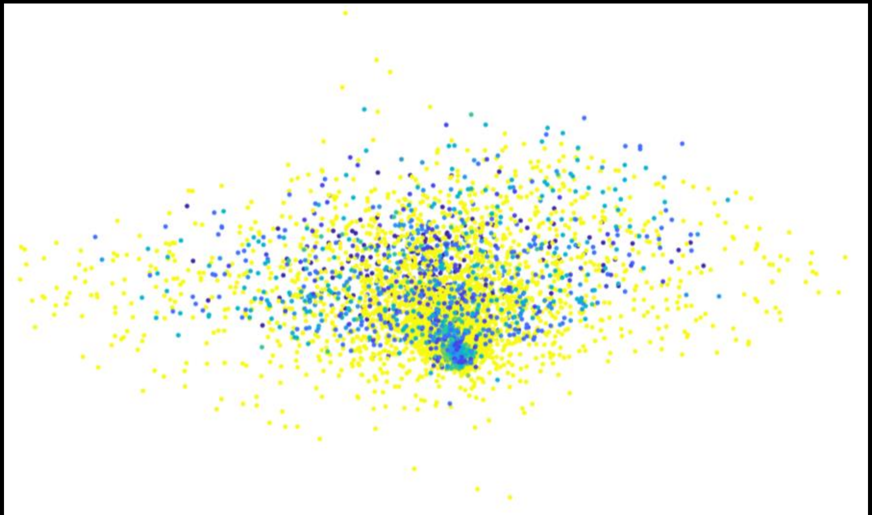
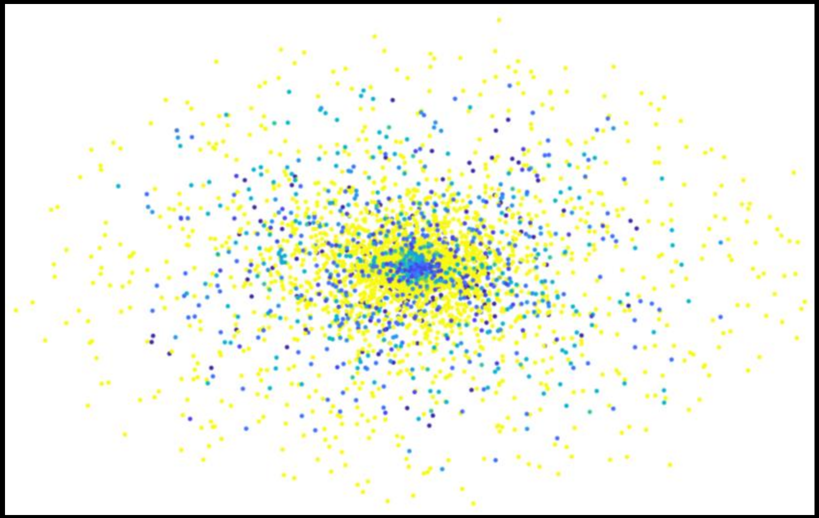
Experiments

PCA on DCT Spectrum



With DC component



Without DC component

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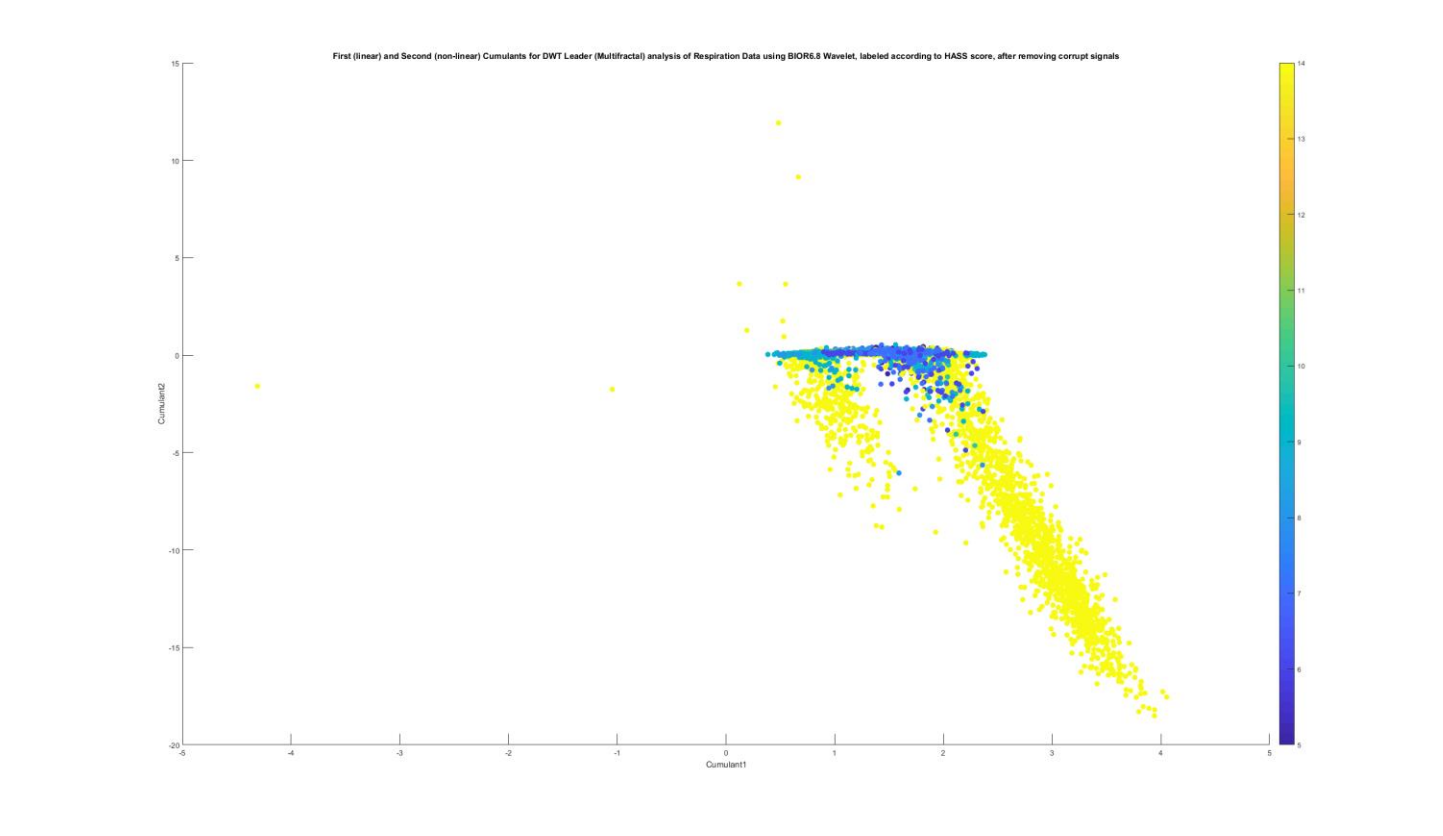
Experiments

First and Second Cumulants on Multifractal Spectrum



- Very clear visual distinction
- Trade-off of frequency-temporal resolution reduces error in both spaces

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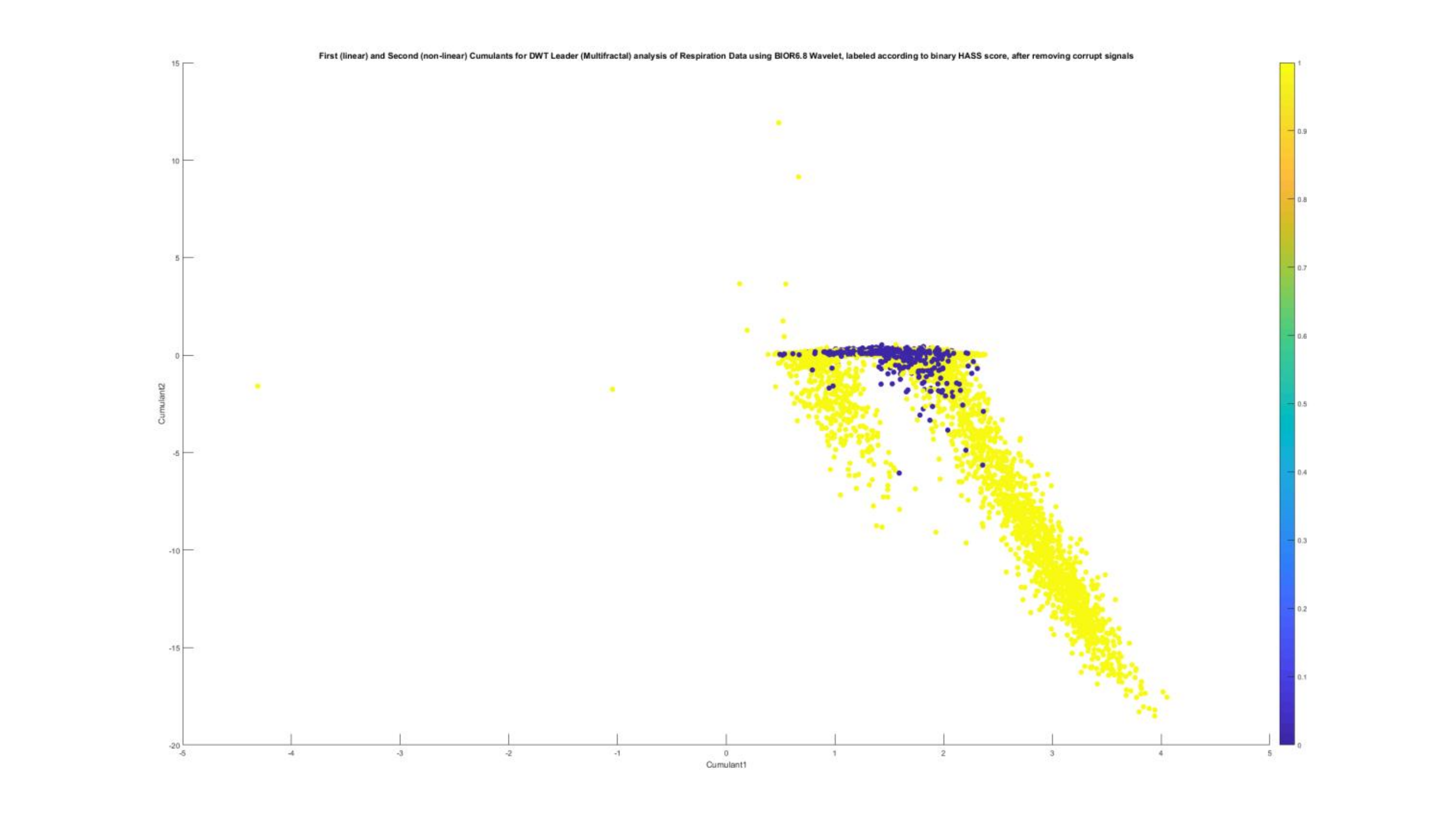
Experiments

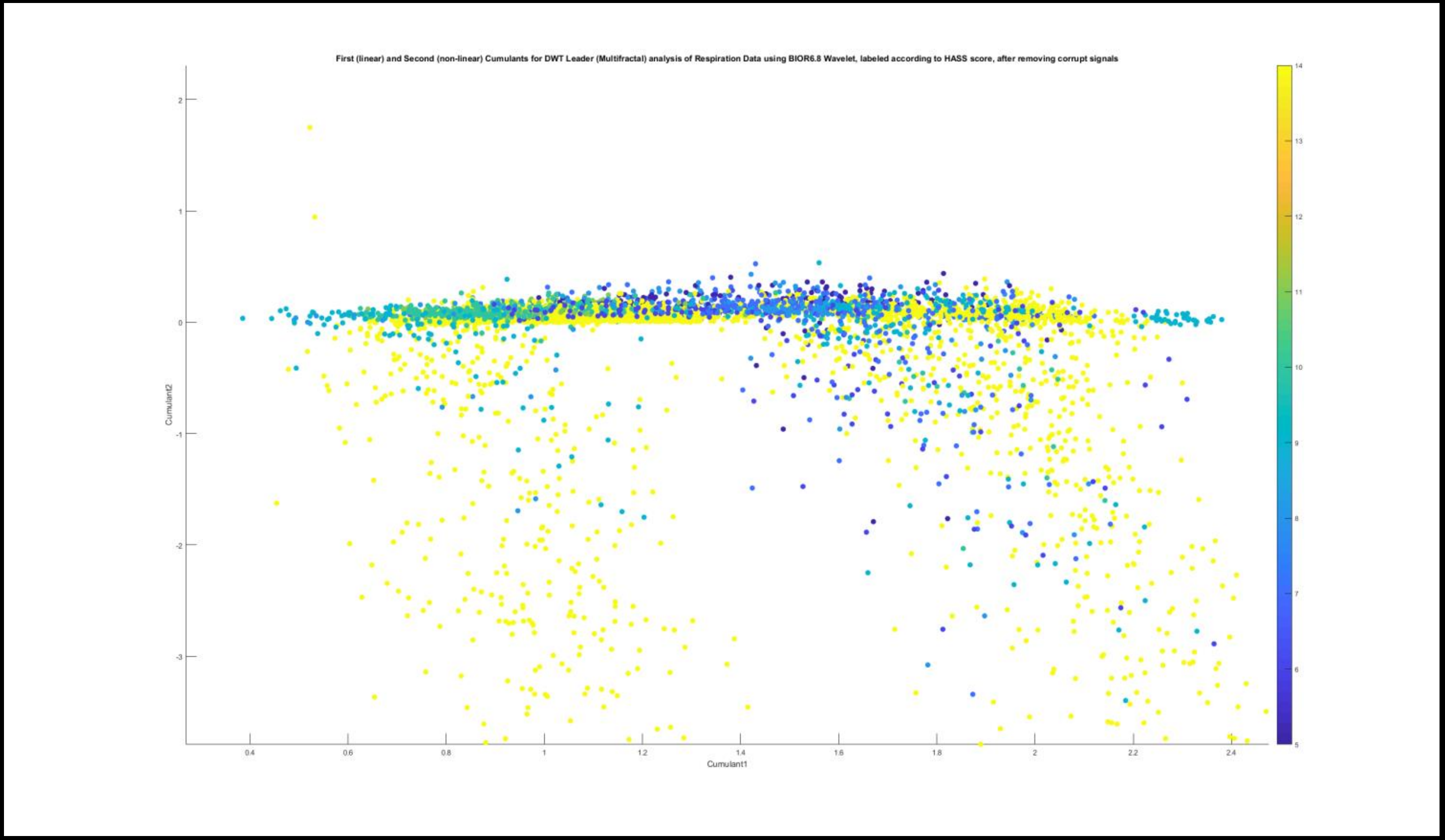
First and Second Cumulants on Multifractal Spectrum



- Very clear visual distinction
- Trade-off of frequency-temporal resolution reduces error in both spaces

Experiments

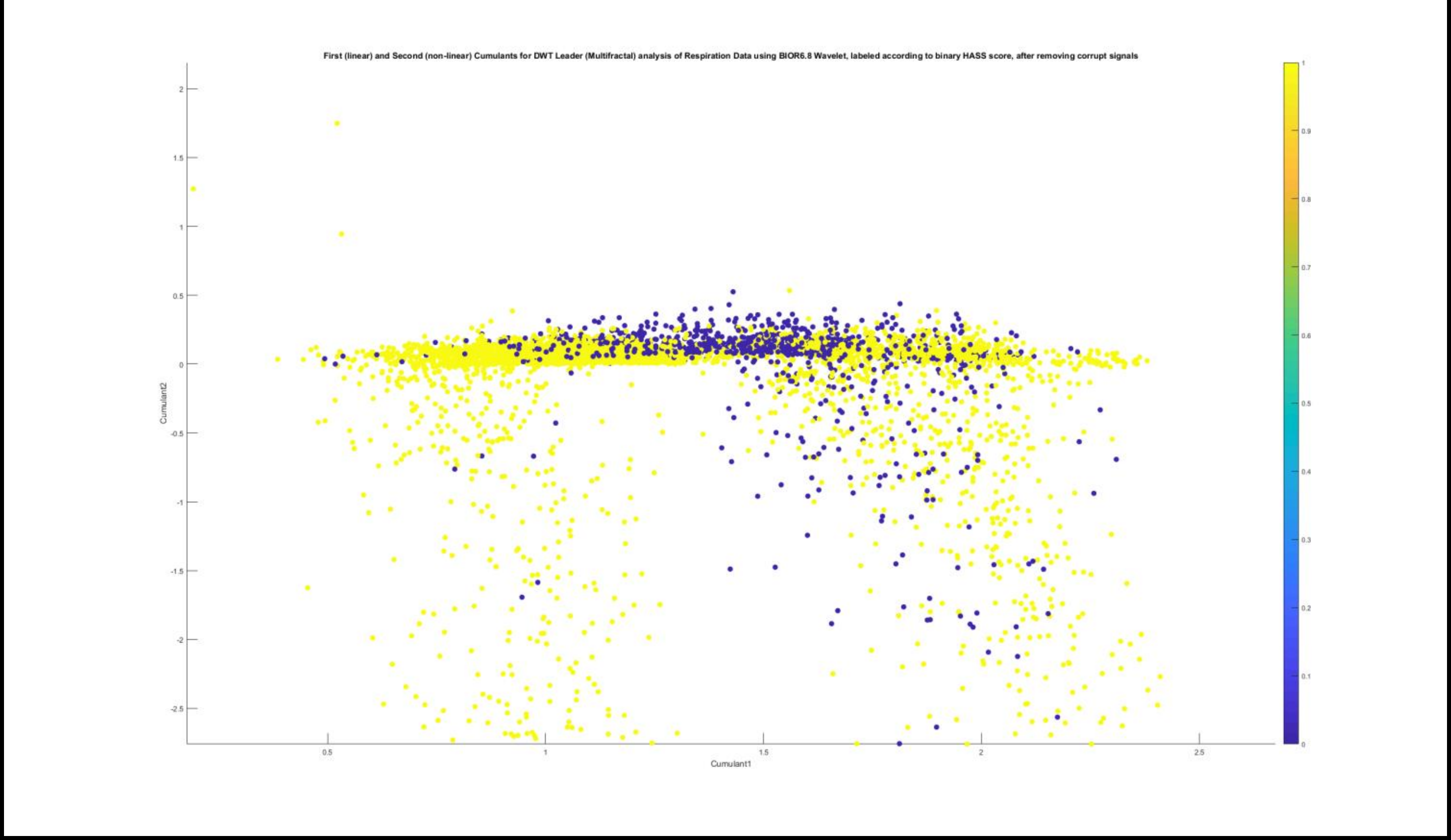
First and Second Cumulants on Multifractal Spectrum (zoom in)



- First cumulant (slope) seems to distinguish between low to medium HASS scores

Experiments

First and Second Cumulants on Multifractal Spectrum (zoom in)



- First cumulant (slope) seems to distinguish between low to medium HASS scores

Conclusions

- Frequency representation does elucidate a clearer HASS distribution
- Multifractal analysis provides us with a very small number (merely 2) of highly interpretable (uncorrelated) features which offer, visually, the most distinction between HASS levels
  - Interesting hypothesis proposed (and confirmed?): An asthmatic fit is an increase in multifracticality of the respiration signal

Next steps

- With such a compact 2D representation of the respiratory waveform, almost any regular ML algorithm can be applied and made to work well (hopefully)
  - Quantifiable (in)validation of hypothesis, rather than just visual
- Can departures from mono to multifracticality in ECG or/and PPG indicate HASS scores as well?
  - If yes, is a combined 4D/6D representation going to be more effective?